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Multivarite Manifold Modeling of Functional Connectivity in Developing Language Networks

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***Abstract*— In a recent paper [2], we presented a method for the modelling of brain networks in the space of symmetric positive definite matrices (Sym$^+$). We showed that this mathematical framework enables an accurate representation of the effects of factors such as age, sex or mental state on the Functional Connectivity (FC) between brain regions.**

I. METHOD

FC of two brain regions is determined from the covariance $\mathrm{Cov}(p_1, p_2)$ of the BOLD fMRI signal time courses p_1 and p_2 observed at distinct locations in the brain. Matrices $P, P_{\alpha\beta} = P_{\beta\alpha} = \mathrm{Cov}(p_\alpha, p_\beta), i, j \in 1 \ldots n$ representing networks of FC between n observed regions are elements of the Riemannian Manifold $\mathscr{M}$ of Symmetric Positive Definite (SPD) matrices Sym_n^+. Positive-definiteness implies $\mathbf{v}^\top P \mathbf{v} > 0 \quad \forall \mathbf{v} \in \mathbb{R}^{+n}, P \in \mathrm{Sym}_n^+$, which renders elements of P interrelated. Euclidean operations do not accurately reflect this underlying geometry of the SPD manifold and can therefore lead to distorted results.

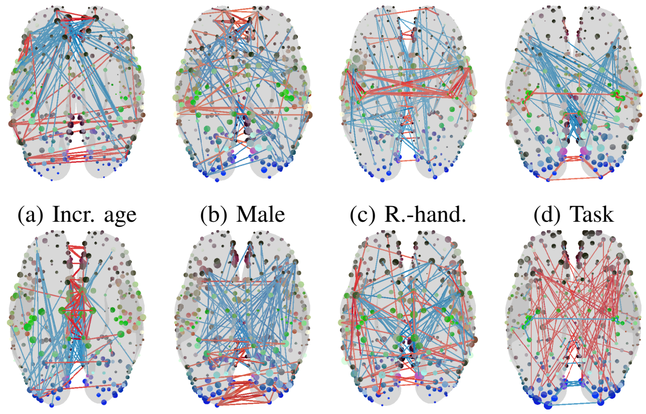
We are interested in describing the effects of known extrinsic information such as patient age, sex or current mental activity on the measured FC matrices. In the Euclidean setting, linear models of the effects of such covariates x_{ij} are fitted to observations P_i obtained from i sources by simple least squares. However, this type of modelling makes the assumption that individual entries $P_{\alpha\beta}$ are mutually independent. By solving the regression model directly in Sym_n^+ [1] as

$$\min_{\tilde{M} \in \mathscr{M}, \mathbf{V}_j \in T_P \mathscr{M}} \sum_{i=1}^{N} \left|\left|\mathrm{Log}_{\tilde{P}_i}(P_i)\right|\right|^2_{T_{\tilde{y}_i}\mathscr{M}}, \quad \tilde{P}_i = \mathrm{Exp}_{\tilde{M}} \left(\sum_{j=1}^{K} \mathbf{V}_j x_{ij} \right) \tag{1}$$

, the resulting intercept $\tilde{M}$ and factors $\mathbf{V}_j$ are by definition elements of Sym_n^+ themselves and therefore capture the interdependence of the entries $P_{\alpha\beta}$.

II. RESULTS

We computed both Euclidean and Riemannian (Eq. 1) models of FC measured in 20 children aged 6 to 13 in relationship to their age, sex, handedness and mental state (at rest vs. performing a language task).



(a) Incr. age (b) Male (c) R.-hand. (d) Task

(e) Decr. age (f) Female (g) Left-hand. (h) Rest

Fig. 1: Effect of varying individual covariates $\mathbf{V}_j$ (from [2])

We were able to show that the Riemannian model (Fig. 1) more accurately reflects the observed population in numerous ways. For example, the expected value of the distribution of the values of the intercept $\tilde{M}$ ($\mathbb{E}[\tilde{M}] = -0.0169$) more closely matches that of the overall population ($\mathbb{E}[M] = -0.0173$) , whereas the Euclidean mean $\hat{M}$ introduces a bias towards anti-correlations ($\hat{M}$ ($\mathbb{E}[\hat{M}] = -0.0418$).

Using both the Euclidean and Riemannian models, we simulated the FC of an average subject and vary the simulated mental state by adjusting the corresponding covariate. We compute the correlation between the simulated FC of a language-specific brain area, the Peri-Sylvian Language area (PSL) and the average FC of the same region obtained from a large reference cohort [3]. The maximum observed correlation between the refrence profile and those obtained from the simulations is higher for the Riemannan model ($R^2 = 0.61$, $\rho < 1\text{e-}37$ compared to $R^2 = 0.58$, $\rho < 1\text{e-}35$), indicating a higher predicitive performance of the Riemannian model.

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